MEASUREMENT OF THE KOSSOVICH NUMBER FOR THAWING FROZEN SOIL

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Successive approximation has been used to consider the thawing (freezing) of moist soil in an unbounded cylinder. A solution has been used in a method for determining the Kossovich number. The measurements agree with calculations.

The Kossovich number

$$\mathrm{Ko} = \frac{L\gamma_f (w - w_n)}{(1 + w) c_T \gamma_T (t_m - t_p)},$$

is an important parameter characterizing the thickness of thawed ground in terms of the thermophysical and other properties, since this includes various physical characteristics. It is difficult to determine each of these singly, so it is convenient to determine them all simultaneously.

The present method for measuring the Kossovich number uses approximate solution of the thawing (freezing) problem for moist soil.

Consider a thin-walled unbounded cylinder containing frozen soil at temperature T_0; at the start, the temperature of the side wall is instantaneously raised to $t_m > t_p$, and then held constant. A layer of thawed soil of thickness $R-\xi$ is produced from the surface, where ξ is the distance from the cylinder axis to the phase-transition boundary. We assume that $t_0 = t_p$ to simplify the problem.

We have to solve the following Stefan's problem for the unbounded cylinder:

$$\frac{\partial^2\theta}{\partial x^2} + \frac{1}{x}\frac{\partial\theta}{\partial x} = \frac{\partial\theta}{\partial \mathrm{Fo}} \tag{1}$$

subject to the boundary conditions

$$\theta|_{x=1} = 1, \tag{2}$$

$$\theta|_{x=\eta} = 0, \tag{3}$$

$$-\frac{\partial\theta}{\partial x}\bigg|_{x=\eta} = \mathrm{Ko}\frac{\partial\eta}{\partial \mathrm{Fo}} \tag{4}$$

and the initial conditions

$$\eta(0) = 1 \tag{5}$$

and this can be obtained by the method of [1]. As our first approximation we take the stationary solution to (1):

$$\theta_1 = 1 - \frac{\ln x}{\ln \eta}. \tag{6}$$

From (6) and (4) we can find [2] the time for complete thawing:

$$\mathrm{Fo}|_{\eta=0} = \mathrm{Fo}^* = \frac{\mathrm{Ko}}{4}. \tag{7}$$

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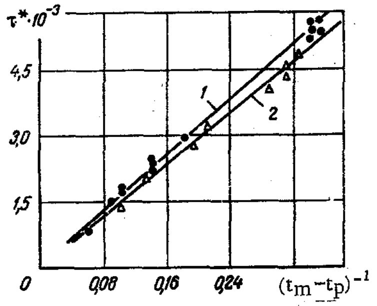

Fig. 1. Thawing time τ^* (sec) as a function of $1/(t_m - t_p)$ ($\deg^{-1}$) for: 1) ice; 2) peat ($w = 5.65$ g/g).

The second approximation is found by substituting (6) with (2) and (3) into (1):

$$\theta_2 = \frac{\eta'}{\eta(\ln\eta)^2}\left[\frac{x^2(\ln x - 1)}{4} - \frac{\ln x}{4\ln\eta} - \frac{\eta^2\ln x}{4} + \frac{\ln x\eta^2}{4\ln\eta} + \frac{1}{4}\right] + 1 - \frac{\ln x}{\ln\eta}. \tag{8}$$

Proceeding similarly, we can find the temperature distribution in the third and subsequent approximations; however, it has been shown [3] that the second approximation will suffice.

When the distribution of (8) has been substituted into (4) and the integration has been performed, we get the time dependence of the thawing boundary:

$$\mathrm{Fo} = \mathrm{Ko}\frac{\eta^2\ln\eta}{2} - \mathrm{Ko}\frac{\eta^2}{4} + \frac{\eta^2}{4} + \frac{1-\eta^2}{4\ln\eta} + \frac{\mathrm{Ko}+1}{4}. \tag{9}$$

The total thawing time Fo* can be found if we put $\eta = 0$ in (9):

$$\mathrm{Fo}|_{\eta=0} = \mathrm{Fo}^* = \frac{\mathrm{Ko}+1}{4}. \tag{10}$$

One can neglect the unity in (10) for large values of the Kossovich number, and it then becomes identical with (7).

Formula (10) applies if t_0 is below the phase-transition temperature by a small amount. Then the amount of heat going to warm the material from t_0 to t_p is 1-5% of the amount needed for the phase transition.

We use (10) to determine Ko by experiment.

In the usual symbols this takes the form

$$\tau^* = A\frac{1}{t_m - t_p} + B, \tag{11}$$

where

$$A = \frac{R^2}{4a_T}\cdot\frac{L\gamma_{f_1}(w - w_n)}{(1+w)c_T\gamma_T}, \quad B = \frac{R^2}{4a_T}.$$

Equation (11) is that of a straight line in coordinates τ^* and $(t_m - t_p)^{-1}$.

Experiment on Ko was performed as follows. The specimen (moist soil) was placed in a thin-walled aluminum cylinder of radius $1.05\cdot 10^{-2}$ m, height $6\cdot 10^{-2}$ m, and wall thickness $2\cdot 10^{-3}$ m. Then the soil was cooled 1-2° below the phase-transition point and kept there for about 4 hours. When the soil had frozen, the cylinder was lowered into a thermostat containing water, which maintained a constant temperature, while mechanical stirring provided for vigorous heat transfer between the liquid and the cylinder. Copper—constantan thermocouples were used to measure the temperatures in the external medium and at the center of the specimen. The output was recorded by an ÉPP—09 pen recorder. The soil-heating curves (from the reading of the thermocouple at the center of the specimen) were used to determine τ^*. The runs were done at various temperatures t_m in the medium.

Figure 1 shows the results for the Kossovich number for ice and peat.

The runs with ice showed that the results agreed to within 5% with the Ko calculated from tabulated data. For peat of water content $w = 5.65$ g/g ($a_T = 1.1\cdot 10^{-7}$ m^2/sec) the Kossovich number can be calculated from

$$\mathrm{Ko} \neq 57\,(t_p - t_m)^{-1}.$$

NOTATION

c_T, γ_T, a_T	are the specific heat, density and thermal diffusivity of thawed ground;
γ_f	is the density of frozen ground;
L and t_p	are the heat and temperature of phase transition;
t_m	is the temperature of medium;
t_0	is the initial temperature;
w	is the moisture content of the ground;
w_n	is the non-frozen moisture;
τ	is the time;
R	is the radius of cylinder;
$x=\frac{r}{R}$, $\eta=\frac{\xi}{R}$, $\theta=\frac{t-t_0}{t_m-t_0}$	are the dimensionless coordinates and temperature;
Fo and Ko	are the Fourier and Kossovich numbers.

LITERATURE CITED

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